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Pearls of Algorithms
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Problem Set 3.3

Problem 1 (Quiz Show)

In a quiz show 100 participants $P_1, \ldots, P_{100}$ can win a trip to Hawaii. The host of the show sets up a room with 100 closed boxes. Every box contains a sheet with a unique number from $\{1, 2, \ldots, 100\}$, and the boxes are placed one after another in a random order in the room. The participants, who can agree upfront on a strategy but cannot communicate anymore once the show has started, enter the room one after another.

The first participant P_1 enters the room first. He can open 50 of the boxes. If none of these boxes contains his own number 1, then all participants lose. Otherwise, he closes all the boxes again and leaves the room. He is not allowed to reorder the boxes or to leave any other hints. Then the second participant P_2 enters the room. He can also open 50 of the boxes. If none of these boxes contains his own number 2, then all participants lose. Otherwise, he closes all the boxes again and leaves the room. He is not allowed to reorder the boxes or to leave any other hints. This is continued until the last participant P_{100}.

Hence, the participants win if and only if each of them opens the box with his own number. If there is a single participant who does not open the box with his own number, everybody loses. Advise the participants on a strategy and compute the probability of winning according to your strategy.

Problem 2 (k-Means Method)

Show that in the worst case local optima of the k-means method can have an arbitrarily bad approximation ratio.

Problem 3 (k-Means++)

We consider the following variant of k-means++: Choose a uniform random subset of size k of the input points and use these k points as initial centers. Show that there are instances on which this variant yields an arbitrarily bad approximation ratio with high probability.

Problem 4 (Coupon Collector's Problem)

There are n different football stickers and one can only buy closed packets, each of which containing one sticker chosen uniformly at random. How many packets do you need to buy in expectation until you have collected all n stickers?